Estimation of the Black-Scholes implied volatility term structure of Equity Warrants listed on the JSE: A comparison of volatility measures

A research report presented to

The Graduate School of Business
University of Cape Town

in partial fulfillment of the requirements for the
Masters of Business Administration Degree

by
Zanoxolo Nkululeko Magadla
December 2000

Supervisor: Professor Haim Abraham
Table of Contents

TABLE OF CONTENTS ............................................................................................................................ II

PREFACE ................................................................................................................................................ IV

ABSTRACT ............................................................................................................................................... V

1. INTRODUCTION ............................................................................................................................... 1
   1.1 BACKGROUND ................................................................................................................................ 2

2. PROBLEM DESCRIPTION ................................................................................................................... 3

3. OVERVIEW OF ACADEMIC LITERATURE ..................................................................................... 5
   3.1 PRE BLACK-SCHOLES .................................................................................................................. 5
   3.2 SUMMARY OF THE BLACK-SCHOLES (1973) MODEL ............................................................. 6
   3.5 MODELLING THE VOLATILITY AS A STOCHASTIC VARIABLE ............................................... 12

4. FORECASTING MODELS .................................................................................................................. 15
   4.1 THE GENERALIZED AUTO-REGRESSIVE CONDITIONAL HETEROSCEDICITY MODEL ....... 15
      (GARCH) ......................................................................................................................................... 15
   4.2 THE EXPONENTIAL GARCH MODEL (EGARCH) .............................................................. 17
   4.3 THE EXPONENTIALLY WEIGHTED MOVING AVERAGE MODEL (EMWA) ........................... 19
   4.4 ESTIMATING THE PARAMETERS ............................................................................................. 19

5. METHODOLOGY ............................................................................................................................... 21
   5.1 INPUT DATA ................................................................................................................................. 21
   5.2 DATA PREPARATION ................................................................................................................ 21
   5.3 THE GARCH(1,1) MODEL ........................................................................................................ 24
   5.4 THE EGARCH METHOD ........................................................................................................... 28
   5.5 THE EWMA METHOD ................................................................................................................ 30
   5.6 SMOOTHING TECHNIQUE .......................................................................................................... 30

6. FINDINGS ........................................................................................................................................... 32
   6.1 THE IMPLIED VOLATILITY ...................................................................................................... 32
   6.2 PERFORMANCE OF THE VOLATILITY MEASURES .................................................................. 32
   6.3 STATIC CONVERGENCE ............................................................................................................ 34
   6.4 SOURCES OF ERROR .................................................................................................................. 34
   6.5 DYNAMIC CONVERGENCE ...................................................................................................... 37

7. CONCLUSION .................................................................................................................................... 39

8. BIBLIOGRAPHY ............................................................................................................................... 41

APPENDIX A: ......................................................................................................................................... 44
   A.1 DETERMINATION OF DAILY RETURNS .................................................................................. 44
   A.2 FORECASTING SHARE PRICES ................................................................................................. 44

APPENDIX B: NORMALITY TESTS FOR DAILY RETURNS ................................................................. 46
Preface

This report is not confidential. It may be used freely by the Graduate School of Business.

I wish to thank Professor Haim Abraham for his valued comments on the process and content of the study. I would also like to thank Professor Trevor Wegner for his assistance with the statistics.

I certify that except as noted above, the report is my own work and all references used are accurately reported.

Signed:

ZANOXOLO NKULULEKO MAGADLA
Estimation of the Black-Scholes implied volatility term
structure of Equity Warrants listed on the JSE:
A comparison of volatility measures

ABSTRACT

The study examines the ability of three volatility-forecasting models to estimate the term structure of implied volatilities. The tests are performed on equity Warrants listed on the JSE with the three measures being the Generalized Autoregressive Conditional Heteroscedicity (GARCH), the exponential GARCH (EGARCH) and the Exponentially Weighted Moving Average (EWMA). The Black-Scholes implied volatility is assumed to reflect the market’s view. The results show that all three methods are unable to forecast the term structure at the 5% significance level but that they are able to indicate future trends. Further, the Black-Scholes model fails to identify an implied volatility under some market conditions thus making benchmarking the models difficult. A conclusion is reached that for the bounds of the study, the three models are not usable if short-term investment decisions need to be taken.

KEYWORDS: Implied volatility, volatility forecasting, Black-Scholes, GARCH, EGARCH, EWMA, term structure, significance level.
1. Introduction

This report is divided into four segments with a total of six core chapters. The first three chapters are essentially introductory. The background section gives a sense of the industry being investigated as it operates in the South African environment. The mechanisms of trading are also briefly touched upon. The next section (problem description) offers the background to the problem to be investigated. Mention is made of past research, which led to this study. It is in this section that the theoretical aim of the research is outlined. This is done through the identification of the hypothesis to be tested. Finally, the literature review attempts to investigate progress in this and other fields of study. Particular emphasis is given to the Black-Scholes option-pricing model since its implied volatility is the benchmark of the whole study.

The second segment (chapter 4) introduces the models to be used in estimating the implied volatility term structure. The generalized forms of the auto-regressive models, termed (p,q) are touched upon but not emphasized. More emphasis is put into discussing the specific forms termed (1,1). The limitations of the models for forecasting beyond a one-day horizon are highlighted and an alternate form introduced. Finally, the optimizing criterion for the various models is introduced.

The third segment gives a sense of the process followed in doing the research. The limitations of the data sources are introduced and the relevant corrective action discussed. This section also deals with the various measures for determining the extent of normality of the company share returns. Further, the mechanism of deriving the benchmark volatility is explained. The various volatility measures are then reintroduced and manipulated to enable programming into a spreadsheet without any random numbers.

The last segment comprises the presentation of the findings of the study. The proposed hypotheses are tested and relevant conclusions drawn. Finally, recommendations for future study are offered.
1.1 Background

The JSE launched the Traded Options Market (TOM) in 1991. This was the first time that Warrants were to be traded in South Africa but the concept did not take off. The second time that Warrants were traded was in 1997 with the listing of Deutsche Bank (DB) Warrants. DB in this instance took the form of a third party on a listed company’s share. Consequently, there was no dilution in the underlying share during exercise of the warrant. The holders of the Warrants also do not participate in dividends. ING Barings and Standard Bank followed as the second and third issuers and later foreign banks issued Warrants. By July 2000, the market was worth R600 million and was dominated by DB.

The Warrants on the JSE can be classified into three categories. These are: share or equity Warrants, index Warrants and Equity basket whose underlying assets are shares, indexes and weighted baskets of shares respectively. The market is dominated by American Calls with some European put option.

One of the primary reasons that Warrants have found acceptance is their gearing and the lower cost of entering the market. Thus, it is possible to participate in an expensive share, like the Anglo shares in the SA market, by paying a fraction of the price for the related Warrant. The price movement in the underlying share is geared up in the Warrant price movement. Thus, if a Warrant was purchased at the same time t as an underlying share, the realized returns after n periods (assuming the market was bullish) are superior for the Warrant. On the downside, the Warrant may not be exercised and thus the losses could be confined to the Warrant purchase price.

The purchase and sale of Warrants is settled through the JSE procedures similar to ordinary shares. This makes trading in this relatively new instrument less daunting for traders that may not be familiar with Warrants. Since the Warrants are issued as American Calls, they can be exercised at any time during their life. This is done through the Bond Exchange of South Africa (BESA). Here, the minimum size of exercise is 50 000 Warrants and additional multiples of 10 000 thereafter. Investors with less that
50 000 Warrants are not allowed to exercise early but are allowed to trade the Warrants on the market.

2. Problem Description

A study by Brooke et al (1999) tested the market price of share Warrants against those calculated using the Black-Scholes option-pricing model (Black-Scholes 1973) as well as Merton’s model (Merton 1973) where the latter allowed for continuously paid dividends. Because there is no dilution effect, JSE listed Warrants were valued as options. Testing at the five percent significance level, they found that in only one out of thirty six possibilities was the difference between the market and the models not significantly different from zero. The greatest overvaluation found was 315%.

One of the underlying hypotheses in the above study is that the equity Warrants can be and indeed are priced using one of the two pricing models. The hypothesis could also be that the Warrant prices are totally market driven and the two models are able to explain this market dynamism. The first hypothesis implies a deliberate course of action by both the traders and issuers of the Warrants while the second a natural drift of prices to the models. Given the small and often illiquid nature of the market, the current study takes the first hypothesis as true i.e. Warrants are priced using the Black-Scholes option pricing model.

In using the Black-Scholes model, several variables need to be uniquely defined. All but one are directly observable on the market and are thus relatively easy to find. On the other hand, the volatility of the underlying share is the one not observable and in fact difficult to define uniquely in time. The Black-Scholes model treats the volatility as time invariant and thus not a significant factor in creating a competitive advantage for forecast accuracy. This is however not the case in reality and modifications to this Black-Scholes assumption are given in more detail later.

This study seeks to explain the market observed volatilities using three forecasting techniques. These methods are the GARCH(1,1), EGARCH(1,1) and the EWMA. To determine the market volatilities, the market price of a Warrant is assumed to be the Black-Scholes price and with everything else taken off the markets, volatility is
calculated to satisfy the Black-Scholes equation. This is the technique used in this study and is often referred to as $C^{-1}(f)$.

The Warrants to be used for the study will be selected as follows.
- The returns of the underlying shares must as far as is possible be reasonably normally distributed.
- Only American call Warrants will be considered
- For any warrant, a period of at least 180 calendar days in its life will be considered
- Whenever possible, the more heavily traded Warrants will take precedence
- Warrants with cover ratios of one and above will be used
- In calculating the implied volatility, the 90 day BA rate will be used as proxy for the risk free rates

To test whether the GARCH, EGARCH and EMWA techniques can forecast market volatilities, the following hypothesis is proposed:

Major null hypothesis: For the equity Warrants listed on the JSE, the GARCH, EGARCH and EWMA techniques each explain the observed market implied volatility at the 5% significance level.

Minor null hypothesis 1: Both GARCH and EGARCH techniques forecast 20 trading day volatilities similarly.

Minor null hypothesis 2: The GARCH and EGARCH each forecast better for data drawn from normally distributed samples.
3. Overview of academic literature

Merton (1973) proved that American options would never be economically exercised before expiration. They (American options) can then be evaluated as European options which are only exercisable at expiry. This assertion is used in this study even though it is not inconceivable for investors to exercise their Warrants before expiry. In that case, Warrants would seem to have been purchased at a premium and this is a disincentive for pre-maturity exercise.

3.1 Pre Black-Scholes

The first analytical attempt at pricing options was by Louis Bachelier in 1900. He used arithmetic Brownian motion to explain the dynamics of the share price, after which the price of a European call option model was derived. This model was criticized by Merton (1973) and Smith (1976) on the basis that it allowed for both negative security and option prices. Further, it did not allow for the time value of money. Given that some of the options have lifetimes up to five years e.g. Warrants, neglecting the time value of money introduces economic errors.

Another attempt to value the options was by Sprenkle (1961), where an assumption was made that the dynamics of the stock are lognormally distributed. The random walk referred to above was assumed to have a drift and thus the negativity of the stock price was ruled out and an allowance for risk aversion allowed for. One of the biggest problems with this model was the allowance for risk aversion. Given that investors have different degrees of risk aversion, it was difficult to derive general as opposed to unique option prices. Attempts to estimate the degree of risk aversion proved unsuccessful.

Boness (1964) presented an improvement on the two models above. This model accounted for the time value of money by discounting the exercise price of the stock using the expected return of the stock.

The models presented above assumed that the option had a similar level of risk to the underlying stock. Samuelson (1965) allowed for a different level of risk. He defined both
the average growth rate of the stock price and that of the option. Thus, the value of the call option increased as its expected average rate of growth increased essential recognizing future prospects in today's price. Samuelson and Merton (1969) proposed a theory where the option price was a function of the underlying stock. Further, the discount rate had to be determined in part by the requirement that the investors hold both the stocks and option to maturity.

3.2 Summary of the Black-Scholes (1973) model

In deriving the model, a stock is assumed to be an underlying asset for the option being priced. The future behaviour of the stock price (which follows a stochastic process) thus impacts directly on the option price. In line with semi-strong form market efficiency, it is assumed that all past information (relevant to the share) is contained in the price of the share today. In predicting the future behaviour of the share price, only its present value is relevant i.e. the share price has the Markov property.

For the purposes of the derivation, general variables will be used to explain some of the processes assumed and finally, a stock specific form will be presented. Assume a variable \(x\) that changes in proportion with time elapsed and with a constant of proportionality \(a\) such that,

\[
\Delta x = a \Delta t
\]

3.1

This however assumes that there is a certainty of a linear change. Now assume that the change also follows a random walk such that the probability of one path is equal to and independent of another.
In other words, assume that the change follows a Wiener process. Denoting this other component $z$,

$$\Delta z = \varepsilon \sqrt{\Delta t}$$

3.2

Where, $\varepsilon$ – random drawing from a standardised normal distribution

Combining the linear and random parts of the change gives a generalised Wiener process for a variable $x$,

$$\Delta x = a\Delta t + b\Delta z$$

3.3

where, the mean of $\Delta x = a\Delta t$

Standard deviation $= b\Delta z/\varepsilon$

Taking the limit as $\Delta t$ tends to zero gives the continuous form of the change,

$$dx = adt + bdz$$

3.3a

Up to now, both $a$ and $b$ have been treated as independent of both $x$ and $t$. The Ito process expresses equation 3.3a (in equation 3.3b) where $a$ and $b$ are functions of the underlying $x$ and $t$ as,

$$dx = a(x,t)dt + b(x,t)dz$$

3.3b

This form can be used for estimating the random change in stock price, $S$,

$$dS = a(S,t)dt + b(S,t)dz$$

3.4

Equation 3.4 seems to suggest that the return $a$ required by the market and the random return $b$ are dependent on both time and the value of the asset. This is obviously
counterintuitive since the cumulative return of parts must equal the return of the sum. The variables $a$ and $b$ are thus defined as,

$$a(S,t) = \mu S$$
$$b(S,t) = \sigma S$$

3.4a

Figure 3.1 gives a simulated share movement using equation 3.4. The share price at time zero was 20 with the volatility and rate of return being 12% and 10% respectively. The trend line was introduced to detect an obvious bias in any one direction but this does not seem to be the case.

**Fig. 3.1 Simulated Share price walk. Period = 365 days**

### 3.2.1 The derivative for underlying stock

Assume a function $G$ defined by the parameters $x$ and $t$. The Taylor series expansion of
the function \( G \) with a normal distribution is give by \textbf{Ito's lemma} as,

\[
dG = \left( \frac{\partial G}{\partial x} \frac{\partial G}{\partial t} + \frac{\partial^2 G}{2 \partial x^2} \right) dt + \frac{\partial G}{\partial x} dz
\]

\[3.5\]

Again, applying this to the stock derivative, whose price \( f \) is assumed to depend on both time held \( t \) and the present value of the stock price \( S \),

\[
df = \left( \frac{\partial f}{\partial S} \mu S + \frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \right) dt + \frac{\partial f}{\partial S} \sigma S dz
\]

\[3.5a\]

Both the changes in the prices of the underlying asset and its derivative have been expressed in equations 3.4 and 3.5a. The \textbf{Wiener} process (\( dz \) term) for both the underlying asset and derivative is the same. Since it is random and unstable, eliminating it will assist in finding closed form solutions for the derivative from equations 3.4 and 3.5a. To eliminate, a portfolio is created so that the \( dz \) coefficients for both equations are equal.

\subsection*{3.2.2 The portfolio}

This is thus,

- Short one option
- Long \( \Delta \) shares such that the value of the portfolio is,

\[
\Pi = -f + \frac{\partial f}{\partial S} S
\]

\[3.6\]
The change in the price of the portfolio in a time interval $\Delta t$ is,

$$\Delta \Pi = -Af + \frac{\partial f}{\partial S} \Delta S$$

3.6a

Both equations 3.4 and 3.5a can be substituted into 3.6 and 3.6a. Consider a hypothetical situation where the portfolio can be shorted and the proceeds used to invest for a time $\delta t$ at a rate $r$. At the end of the time period, the investment yields an interest income of $r\Pi \delta t$. If this (income) is greater than the change in the value of the portfolio (equation 3.6a), an arbitrage opportunity exists. To ensure that this does not occur, the portfolio is assumed to yield at the risk free interest rate such that,

$$\Delta \Pi = r \Pi \delta t$$

3.6a(i)

Solving this equity gives the Black-Scholes differential equation,

$$\frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{\sigma^2 S^2}{2} \frac{\partial^2 f}{\partial S^2} = rf$$

3.7
3.3 Shortcomings of the model

3.3.1 No arbitrage argument

Here the assumption is made that rehedging is continuously being done. In reality, this is not the case since rehedging can only be discrete. Consequently, a rehedge error arises and equation 3.6a(i) does not hold. To minimise this hedge error, Wilmott (1998) constructs a delta-hedged portfolio as in the Black-Scholes.

\[ \Pi = f - \Delta S \]

3.8

Since the analysis is now discrete, he uses the Taylor series expansion to derive \( \delta \Pi \) as a power series. Using the following assumptions,

- Choose \( \Delta \) to minimise the variance of \( \delta \Pi \)
- Value the option by setting the expected return on \( \Pi \) equal to the risk free rate.

This introduces an unstable term to the Black-Scholes differential equation in the form,

\[ \frac{1}{2} (\phi^2 - 1) \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} \]

3.8a

The Black-Scholes differential equation then changes to,

\[ \frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + \frac{1}{2} (\phi^2 - 1) \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf \]

3.9

In this case, the closed form solution is hardly possible and a numerical solution would be more adequate.
3.5 Modelling the volatility as a stochastic variable

The Black-Scholes model assumes that the volatility of the underlying asset is constant. This is convenient but a reality check confirms that this is not the case. The volatility smiles indicate that the volatility of the underlying is actually stochastic. It can be shown that the price of a European option on a stock is equal to the Black-Scholes price integrated over the probability distribution of the average variance rate if the stock price and its volatility are instantaneously uncorrelated (Hull-White (1996)).

Hull-White (1996) gives the following analysis for the stochastic volatility problem.

Consider a derivative with an underlying asset $S$ and instantaneous variance $V = \sigma^2$ and which obeys a stochastic process;

\[
dS = \phi S dt + \sigma S dw \\
dV = \mu V dt + \xi V dz
\]

where $\phi$ may depend on $S$, $\sigma$ and $t$, $\mu$ and $\xi$ may depend on $\sigma$ and $t$. The correlation of the Wiener processes is $\rho$.

To determine the differential relationship between the security $f$ that depends on state variables $\theta_i$, the expression by Garman (1976) was used such that;

\[
\frac{\partial f}{\partial t} + \frac{1}{2} \sum_{i,j} \rho_{ij} \sigma_i \sigma_j \frac{\partial^2 f}{\partial \theta_i \partial \theta_j} - rf = \sum_i \theta_i \frac{\partial f}{\partial \theta_i} (\mu_i + \beta_i (\mu^* - r) )
\]

where $\sigma_i$ is the instantaneous standard deviation of $\theta_i$, $\rho_{ij}$ is the instantaneous correlation between $\theta_i$ and $\theta_j$, $\mu_i$ is the drift rate of $\theta_i$, $\beta_i$ is the vector of multiple regression betas for the regression of the state variable
returns on the market portfolio and the portfolios most closely related with the state variables,

$\mu^*$ is the vector of instantaneous expected returns on the market portfolio and $r$ is the vector with elements that are the risk free rate $r$.

For the problem being considered, the two state variables are the stock price ($S$) and the variance of the returns ($V$). Substituting these into equation 3.11 and solving gives the differential equation of the form of the Black-Scholes differential except that in this case the volatility is assumed stochastic.

$$\begin{align*}
\frac{df}{dt} + \frac{1}{2} \left( \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + 2 \rho \sigma \xi S \frac{\partial^2 f}{\partial S \partial V} + \xi^2 V^2 \frac{\partial^2 f}{\partial V^2} \right) - rf &= \\
&- rs \left( \mu - \beta_V (\mu^* - r) \right) \sigma^2 \frac{\partial f}{\partial V}
\end{align*}$$

3.12

where $\rho$ is the instantaneous correlation between $S$ and $V$,

$\beta_V$ is the vector of multiple regression betas for the regression of the variance returns on the market portfolio.

If it is assumed that the equation 10 is a process for $V$ in a risk neutral world, equation 3.12 then simplifies to,

$$\begin{align*}
\frac{df}{dt} + \frac{1}{2} \left( \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} + 2 \rho \sigma \xi S \frac{\partial^2 f}{\partial S \partial V} + \xi^2 V^2 \frac{\partial^2 f}{\partial V^2} \right) - rf &= \\
&- rs \sigma^2 \frac{\partial f}{\partial V}
\end{align*}$$

3.12a

Equation 3.12a can be solved numerically to calculate the price of a derivative (in this case a warrant). Alternatively, an analytical solution can be sought by using the risk neutral valuation method similar to that for the Black-Scholes differential equation.
The price of the option is finally given as,

\[ f(S_t, \sigma_t^2, t) = e^{-r(T-t)} \int f(S_T, \sigma_T^2, T) p(S_T \mid S_t, \sigma_t^2) dS_T \]

3.13

where \( T \) is the time to maturity of the derivative,
- \( S_t \) is the value of the stock at time \( t \),
- \( \sigma_t \) is the instantaneous standard deviation at time \( t \),
- \( p(S_T \mid S_t, \sigma_t^2) \) is the conditional probability distribution of \( S_T \) in a risk neutral world given the derivative price and variance at time \( t \), and
- \( f(S_t, \sigma_t^2, T) \) equals \( \max\{0, S - X\} \).

Using equations 3.12a and 3.13, a more accurate value of the option price can be obtained if it is assumed that the variance assumption in the Black-Scholes model leads to the biggest proportion in the error on the price.
4. Forecasting models

The three models are outlined below.

4.1 The Generalized Auto-Regressive Conditional Heteroscedicity Model (GARCH)

Researchers like French, Schwert and Stambaugh (1987) have observed that the variance of stock returns changes over time. On the other hand, pricing models such as the Black-Scholes are based on the assumption that the variance is continuous and non-stochastic. There is thus a need to estimate the future volatilities of underlying assets in order for efficient pricing to occur.

The GARCH model is one such model. Bollerslev (1986) proposed a variant of this model as a tool for forecasting future econometric volatilities. In its general form, the model is of the form GARCH (p,q) where the instantaneous variance is calculated from the most recent p market return observations and the most recent q observations of the variance rate. Bollerslev’s model however uses the most recent observations of both the variance rate and the stock returns hence the notation GARCH (1,1). This is the model used in this study. For this model, the stock return and its volatility are expressed as:

\[ \text{Ln} \left( \frac{S_t}{S_{t-1}} \right) = \mu + \xi_t \sigma_t, \]

4.1

And

\[ \sigma_t^2 = \beta_0 + \beta_1 \xi_{t-1}^2 + \beta_2 \sigma_{t-1}^2, \]

4.2

where: \( \mu \) - mean stock price return
\( \xi_t \) – Gaussian White noise and drawn from N(0,1)
\( \sigma_t \). Stock return volatility
\( \varepsilon_t = \sigma_t \xi \)
\( \beta_i \) - time independent parameters
Equation 4.2 assumes that the variance rate oscillates about a common mean as given by the variance rate V (equal to \( \beta_0 / (1 - \beta_1 - \beta_2) \) i.e. there are mean reversion properties. Further, the weight given to the immediate past variance rate is not necessarily the same as that of the corresponding daily returns.

This assumption of mean reversion is contrary to studies performed by Pindyck (1984) and Poterba and Summers (1986). According to these studies, empirical evidence showed that the assumption of constant conditional means and variances for returns on stocks was unrealistic. If this were taken to be true, then the average variance rate would change with time and thus the weights \( \beta_k \) as well. This is contrary to the assumption of time independence as given in the GARCH (1,1) literature.

The model is useful in estimating volatility over a one-day horizon. Thus, it is not possible to take longer term trading positions on the market based on the estimates derived. Heynen, Kemna and Vorst (1994) suggested a model for extending the GARCH (1,1) model to forecast average expected volatilities to the expiry of an option. By definition, the average expected volatility to maturity for an option is the mean of the daily volatilities in this period. For the GARCH (1,1) model, this was given as an arithmetic mean

\[
\sigma^2_{m,v}(t,T) = \frac{1}{T} \sum_{k=1}^{T} E_t(\sigma^2_{r,k})
\]

where \( T \) is the time number of periods from \( t \) to the maturity date and \( E_t \) the conditional expectation operator at \( t \).

It can be deduced from equation 4.3 that the instantaneous and expected volatilities converge towards maturity and are in fact identical on the maturity date. By induction and using the independence of \( \xi_k \) it was shown that

\[
E_t\left[\left(\frac{\xi_k^2}{\tau_{r,k}}\right)\right] = \beta_0 (1 + \sum_{m=1}^{k-1} (\beta_1 + \beta_2)^m) + (\beta_1 + \beta_2)^{k-1} (\beta_2 \xi_t^2 + \beta_2) \sigma_t^2
\]

4.4
Substituting equation 4.4 into 4.3, an expression can be found for the average expected volatility. For any two time period to maturity \( (T_1 \text{ and } T_2) \), it can be shown that

\[
\left[ \sigma^2_{\Delta t} (t, T_i) - \bar{\sigma} \right] = \frac{T_2}{T_1} \gamma \frac{\bar{\sigma}}{\bar{\sigma} - 1} \left[ \sigma^2_{\Delta t} (t, T_2) - \bar{\sigma} \right]
\]

where, \( \sigma^2 = \beta_1/(1-\beta_1-\beta_3) \) is the long run variance rate,
\( \gamma = \beta_1 + \beta_2 \),
\( T_1 > T_2 \).

It is thus possible to obtain a closed form solution from equation 4.5 since the random term has been eliminated.

### 4.2 The Exponential GARCH Model (EGARCH)

The observation that stock returns are negatively correlated with the changes in the volatility \( \text{(Black 1976 and Christie 1982)} \) led to the development of the EGARCH model \( \text{\textit{(Nelson 1991)}} \). The stock return and volatility are expressed as:

\[
\ln\left( \frac{S_t}{S_{t-1}} \right) = \mu + \xi \sigma_t
\]

and

\[
\ln \sigma_t^2 = \beta_0 + \beta_1 \ln \sigma_{t-1}^2 + \beta_2 \xi_{t-1} + \beta_3 (|\xi_{t-1}| - \frac{2}{\sqrt{\pi}})
\]

where the parameters are as defined in the presentation of the GARCH model. The weights are time independent thus rendering the long run variance rate also constant. The assumption of mean reversion therefore is upheld in this model.

The average expected volatility for the remaining time to maturity is assumed to be the geometric mean of the daily instantaneous volatilities in the period. This is justified by
work that suggested that there is a close resemblance between the geometric average expected volatilities and the implied volatilities of the theoretical option prices (Heynan et al 1994). This average is thus expressed as

$$Ln\sigma_{av}^2(t, T) = \frac{1}{T} \sum_{k=1}^{T} LnE_i[\sigma_i^2]$$

4.7

Using equation 4.6, and evaluating the expected instantaneous variance, a relation is established between average expected volatilities at different times (T1 and T2) to expiry. This is expressed as

$$[Ln\sigma_{av}^2(t, T_1) - Ln\sigma_{av}^2(t, T_2)] = \frac{T_2}{T_1} \cdot \frac{1 - \beta_1}{1 - \beta_1^T} \cdot [Ln\sigma_{av}^2(t, T_2) - Ln\sigma_{av}^2(t, T_1)] + R$$

4.8

where,

$$\sigma = \exp\left[\frac{\beta_0 - \beta_1}{1 - \beta_1} \cdot \sqrt{\frac{2}{\pi}} + \frac{1}{2} \left(\beta_2 + \beta_3^2\right) \right] \cdot \prod_{m=0}^{\infty} \left[ F_m(\beta_1, \beta_2, \beta_3) + F_m(\beta_1, -\beta_2, \beta_3) \right]$$

4.8 (a)

and

$$F_m(\beta_1, \beta_2, \beta_3) = N[\beta_1^{\alpha m} (\beta_3 + \beta_2)] \exp[\beta_1^{\alpha m} \beta_2 \beta_3]$$

4.8 (b)

Equation 4.8 differs from the corresponding GARCH (1,1) not only by its logarithmic nature but also by the presence of a constant R. However, for periods of more than about one month between the two average expected volatilities, it (R) is close enough to be neglected. Thus, for the purposes of this study, R is assumed equal to zero.
4.3 The Exponentially Weighted Moving Average Model (EMWA)

This method is useful in estimating daily volatilities. Here, the weights assigned to historical volatilities decrease exponentially as one moves back in time. The formula for this is given as:

$$\sigma^2_n = \lambda \sigma^2_{n-1} + (1-\lambda)u^2_{n-1}$$

4.9

The equation impounds both historical as well as the most current information on the underlying stock's volatility. The immediate past volatility serves to keep the forecasted volatility in line with the trend established over time. On the other hand, the returns indicate the most recently observed changes in the market. Thus, a large weight ($\lambda$) implies that very little significance is placed on the most recent market trends and the response to new information to the market is very slow. On the other hand, a very low weight means that the forecasted volatility will be highly volatile due to the Markov property of the stock returns.

Unlike the other models presented, this model requires that very little data is stored since only the most current estimates are used. In addition, the number of parameters to be estimated is restricted to the weight ($\lambda$) only. To estimate the weight, the implied volatility of a warrant can be observed over a period of time and the EWMA model can then be fitted. This technique is being used in the financial services industry (Hull 2000), with companies like JP Morgan estimating it at around 0.94 based on a sample of data spanning 25 days.

4.4 Estimating the Parameters

In all of the models described, the weights are not directly observable in the market place. The technique used to determine these weights is the maximum likelihood method. This is based on the probability distribution of the returns occurring in the order that they are observed.
In both the GARCH and EGARCH, the likelihood function that was maximized took the form

\[
L = \sum_{i=1}^{m} \left[ -\ln(\sigma_i^2) - \frac{u_i^2}{\sigma_i^2} \right]
\]

with \( m \) – time periods

\( \sigma_i^2 \) - market variance

\( u_i \) - stock return.

\( L \) is maximised by continuously changing the weights i.e. changing the relative influences of the historical variances and the current share returns. Thus, the maximum value represents the optimum weighting.

In a stable share market, a constant likelihood function could hold for extended periods without a need for readjustment. If there is a significant movement in the market (in any direction), the most recent observation of the variance rate will progressively explain less of the observed volatility. Instead, more attention needs to be given to the current market signal, in this case the daily returns on the shares. In other words, the weighting should shift towards the daily returns essentially upsetting the established likelihood value. A new likelihood function therefore needs to be calculated.
5. Methodology

The input data used in the study as well as the preparation thereof are presented in this section. Since the study is sample based, the selection criterion (for the sample) is discussed. Finally, the application of the three different volatility models is outlined and underlying assumptions discussed.

5.1 Input Data

The market data used in the study consists of 743 (19 July 1997 to 8 September 2000) daily share prices and dividends. Equity Warrants issued with these shares as underlying assets were then selected such that they had been trading for at least 180 calendar days by the 8th of September 2000. Using these dates, 43 shares were identified (appendix B). For both the Warrants and the underlying shares, only the closing prices were considered. Further, the dividend yield was used as proxy for the assumption that dividends on a share are continuously paid. The 90-day Bankers Acceptance (BA) rate was used as proxy for the risk free rate and in line with the respective Warrants, daily rates were used. Initially, the data was sourced off McGregors Raid but it was later discovered that the database was flawed since it included some public holidays as trading days. A correction was thus made to some of the data using the INET Bridge database.

5.2 Data Preparation

The primary focus of the study was using the Black-Scholes implied volatility as the benchmark for testing the other volatility forecasting techniques. As already discussed, the Black-Scholes option-pricing model assumes that the returns on the underlying share prices are log-normally distributed. The daily underlying shares were adjusted for the expected dividend stream i.e. the value of the share was taken as the current trading price less the present value of future dividend cash flows to a one year horizon.
This is illustrated as:

\[ S_t = S_0 e^{-dy} \]

5.1

where \( dy \) = dividend yield

Having determined the adjusted share prices, daily continuously compounded returns were then calculated as follows.

\[ u_t = \frac{1}{n} \log \left( \frac{S_t}{S_{t-n}} \right) \]

5.2

where, \( n \) – number of days over which compounding occurs (see Appendix A for the derivation).

Despite the fact that it did not make any significant difference, the factor \( n \) was introduced into the returns to recognize the lower than average returns on days following non-trading ones e.g. Mondays.

5.2.1 Test for normality

The returns so determined were then used to test for normality of the population from which the sample (of share prices) was drawn. Here, three measures were used and these include the Kolmogorov Smirnoff and the Jarque Bera tests for normality. Although the Jarque Bera test impounds both the skewness and the Kurtosis of a distribution, an investigation of the skewness was used as a third measure. The hypothesis testing was set up as follows.
\textit{H}_0: \ The data are normally distributed
\textit{H}_1: \ The data are not normally distributed.

The test was conducted at the 5\% significance level i.e. \( \alpha = 0.05 \). For the Jarque Bera test, the sample sizes were considered large enough. Thus the test for normality was conducted for a Chi-Squared distribution with two degrees of freedom. At this level of significance, most of the returns tested led to the acceptance of the alternate hypothesis in the case of the Jarque Bera test. For the Kolmogorov Smirnoff test, all of the returns were found to confirm the alternate hypothesis.

Despite this finding, the Warrants and other options are in reality priced using the Black-Scholes option-pricing model although this model assumes normal distribution. A decision was thus taken to continue with the study and some outliers were eliminated through the use of the Skewness test. A Skewness benchmark for the usability of data in the study was set as -0.7 <Sk< 0.7. Nine share returns were found to have Skewness outside the benchmark setting and these were then rejected from the study.

The study was thus performed with essentially non-normal data but this is not atypical of what occurs in reality. Therefore, useful insights can be gained on the behavior of the market variables from the study. The results of the tests are summarized in appendix B.

5.2.2 Implied Volatility

This was calculated by inverting the Black-Scholes option-pricing model with the volatility of the share as the only unknown. The actual values of the risk-free rate, share price and exercise price for the specific day were used. The calendar convention was arbitrarily adopted as 365 days per year instead of say 360 with the market price of the Warrants being used as the target price in determining the implied volatility. To this end, the Solver Optimization package was used. To avoid manually repeating the Solver routine over 4000 times, code was written in Visual Basic to interface Solver with the Excel spreadsheet. This program is included in appendix C.
Clearly, the implied volatility of the underlying asset does in reality exist even if the warrant has stopped trading. To uniquely define this however requires that the warrant price is known with certainty. Thus that those Warrants that had stopped trading within the period under discussion were excluded from the study.

5.2.3 Other problems

Some of the Warrants were issued and then later withdrawn earlier than their expiry dates. Here it is assumed that these Warrants were very thinly traded or were in fact never extensively taken up. These Warrants were thus excluded from the study. Finally, the study was conducted on twenty-one company daily returns.

5.3 The GARCH (1,1) Model

Equation 4.3 gives the relationship between the average expected volatility at time $T_1$ as a function of that at time $T_2$ where $T_1 > T_2$. The out of the ordinary movements in the markets are not impounded since all of the parameters can in fact be estimated at the beginning of the life of the warrant and then theoretically used to the end of the life of the warrant. To correct this, equation 4.2, which impounds both historical trends and current market movements, is used to estimate the instantaneous volatility on the day of the computation. This (instantaneous volatility) is then used as a seed in approximating the average volatility at a future date by setting $\sigma_{T_2} = \sigma_{av.}(t,T_2)$ and then calculating $\sigma_{av.}(t,T_1)$.

5.3.1 The instantaneous volatility

Both equations 4.1 and 4.2 in section 4.1 include a random term in defining the dynamics of the returns and instantaneous volatilities respectively. Thus, it is not possible to have a close form solution since the random numbers give a spread of possibilities and not a unique solution. It was thus important for the purposes of this study that the random factor was removed from both equations in order for it to be possible to model on the Excel spreadsheet.
This was done as follows:

\[ \xi_{t-1} = \frac{1}{\sigma_{t-1}} \left( \ln \left( \frac{S_{t-1}}{S_{t-2}} \right) - \mu \right) \]

5.3

where, \( \varepsilon_t = \sigma_t \xi \) and thus,

\[ \sigma_t^2 = \beta_0 + \beta_1 \left( \ln \left( \frac{S_{t-1}}{S_{t-2}} \right) - \mu \right)^2 + \beta_2 \sigma_{t-1}^2 \]

5.4

The parameters for estimating the instantaneous volatility can now be uniquely defined and thus easily programmable. The modified equation (from equation 4.3) for the average expected volatility that was used for the exercise is:

\[
\left[ \sigma_{\Delta t}^2 (t, T) - \sigma \right] = \frac{T_1}{T_1} \left[ \frac{\gamma_1^2}{\gamma_2^2} - 1 \right] \left[ \sigma_{\Delta t}^2 (t, T) - \sigma \right]
\]

5.5

### 5.3.2 Estimation of the volatility weights

To find the values of \( \beta_0, \beta_1, \beta_2 \) the maximum likelihood function given in equation 4.8 is used. The variance is the subject of the volatility-updating scheme in GARCH (1,1) and equation 4.8 is then expressed as:

\[ L = \sum_{i=1}^{n} \left[ -\ln(\sigma_i^2) - \frac{u_i^2}{\sigma_i^2} \right] \]

The volatilities in the likelihood function are updated by changing the weights in the instantaneous volatility equation until the value of L is maximized.
The different weights are locked in relation to each other by the following constraints:

\[ \beta_1 + \beta_2 < 1 \]
\[ \sigma^2 = \beta_0/(1 - \beta_1 - \beta_2) \]

Two approaches could thus be taken in determining the long run average variance, these being: allow the weights to fluctuate until the maximum value of L is found and thus that of the long run average variance or use variance targeting. The former is self-explanatory and the latter will be discussed below.

### 5.3.3 Variance targeting

This method entails taking the arithmetic or geometric average of the historical variances as exhibited by the returns on the shares. The value so obtained is then used as a target ratio. The weights are changed such that this variance is attained at the same time as the maximum value of L. Clearly this approach reduces the iteration time. Further, the chances of a local maximum value of L being identified by the optimization package as the maximum are also reduced. This method is thus ideal for general-purpose optimization software such as Solver in Excel.

There are however serious problems of timing with this method. A decision needs to be taken on the period over which the variance is to be estimated. In fact, the method of calculating past variances should itself be defined. To solve the time problem, an assumption could be taken on the period of the mean reversion of the share returns and this could then be used as the ideal time frame. The implied volatilities over this time frame could then be used to find the corresponding variances.

The warrant market on the JSE is relatively young. Consequently, most of the underlying shares have the Warrants being analyzed as the first set to be issued. There is thus no
basis for calculating the historical implied volatility. Thus, the long run variance cannot be defined in this manner.

For the purposes of this study, it was thus decided that the average variance would depend entirely on the weights and would not be fixed. For the purposes of programming, a condition was set such that the variance was non-negative.

5.3.4 The average returns $\mu$

The mean share price return was taken as the average of the past returns to about one year. It might be argued that this is erroneous since the historical returns also impound the random nature of these returns. However, this random term is supposed to be normally distributed and over time would thus tend to move as much on the positive side of the mean as the negative. It is thus assumed that if a sufficiently long horizon is taken, these random terms will tend to cancel out and therefore a clean average return can be found. Ideally, this value would be expected to be positive and this was the case for most of the shares under consideration. A negative value would indicate an extreme volatility biased towards the negative side of the mean.

5.3.5 Spreadsheet layout

The following is an example of the layout of the spreadsheet as used. To initialize the instantaneous volatility column (column five) an assumption is made that this is equal is to square of the calculated daily returns $u_i$. To avoid this assumption, calculations can be started from the 11 / 12 / 99, but this only solves the problem of losing one data point since it too needs to be initialized.
<table>
<thead>
<tr>
<th>Date</th>
<th>Day i</th>
<th>S_i</th>
<th>u_i</th>
<th>( \sigma_i^2 )</th>
<th>L_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 / 11 / 99</td>
<td>1</td>
<td>400</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13 / 11 / 99</td>
<td>2</td>
<td>410</td>
<td>0.024693</td>
<td>0.000523</td>
<td>6.368</td>
</tr>
</tbody>
</table>

Table 5.1: Sample spreadsheet layout for parameter estimation

The subsequent values of \( \sigma_i \) are calculated by changing the weights and recognizing the daily returns.

The weights determined in this way are assumed to be time independent for the time frame of the study. This assumption thus infers that the expected volatilities have mean reversion properties with a constant mean determined by the weights.

### 5.4 The EGARCH method

The equation used in estimating the average future volatility was determined using equation 4.4 in section 4.5. Again the instantaneous volatility as well as the daily continuous returns are expressed in terms of random numbers. Similar to the GARCH (1,1) case above, this means that a closed form solution is unattainable. The random element is thus removed as follows: The daily returns are expressed in a similar manner for both the GARCH (1,1) and this model. Substituting for the Gaussian white noise (\( \xi \)) in equation 4.2 yields:

\[
Ln\sigma_t^2 = \beta_o + \beta_1 Ln\sigma_{t-1}^2 + \beta_2 (Ln(\frac{S_{t-1}}{S_{t-2}}) - \mu) + \beta_3 (|Ln(\frac{S_{t-1}}{S_{t-2}}) - \mu| - \frac{2}{\sqrt{\pi}})
\]

Again the instantaneous volatility is then used as a reference point when estimating the future average expected volatility.
Substituting the above equation into equation 4.5 in section 4.2 gives:

\[
\left[ \ln \sigma^2_{\alpha}(t,T_t) - \ln \sigma^- \right] = \frac{T_2 - T_1}{T_1} \frac{1 - \beta_1^2}{1 - \beta_1^2 T_1} \left[ \ln \sigma^2_{\alpha}(t,T_2) - \ln \sigma^- \right]
\]

5.7

The current state of the market is thus impounded in the estimation of future volatilities.

5.4.1 Estimation of the weights

The maximum likelihood function given in equation 4.10 is again used to find the values of the weights \(\beta_0, \beta_1, \beta_2, \beta_3\) used. Although the instantaneous volatility is expressed in log form, it was assumed that the variance obtained is again the subject of the updating scheme in EGARCH. Thus, the probability of the returns occurring as they are observed is represented by the likelihood function below,

\[
L = \sum_{i=1}^{m} \left[ -\ln(\sigma^2_i) - \frac{\ln^2 (\sigma_i)}{\sigma_i^2} \right]
\]

By varying the four different weights, the value of the likelihood function was altered until a maximum value was found. The different weights are locked in relation to each other by the following constraints:

\[
\beta_1 + \beta_2 < 1
\]

and,

\[
\sigma = \exp \left[ \frac{\beta_0 - \beta_1 \sqrt{2}}{1 - \beta_1} \sqrt{\frac{2}{\pi}} + \frac{1}{2} \left( \beta_2^2 + 1 - \beta_1^2 \right) \right] \prod_{n=0}^{\infty} \left[ F_n(\beta_1, \beta_2, \beta_3) + F_n(\beta_1, -\beta_2, \beta_3) \right]
\]
For the same reasons as the GARCH (1,1) section, the average long run volatility was not used as a target for the weights. Instead, it was allowed to reach an equilibrium position at the same time as the likelihood function. It was then assumed that this is level of mean reversion of the average expected volatilities valid for the duration of the study. As was expected, the second part of the average volatility was found to be a convergent series. For all the shares under consideration, this point (of convergence) was found to be one.

5.5 The EWMA Method

Similar to the above models, this model was initialized using only the daily returns for the first day of computation. The model was however the easiest to apply since only one weight had to be estimated. In this case, a weight of 0.94 was applied to the historical volatilities and most current returns as estimated by J.P. Morgan. The weight was then fixed for the remainder of the time and used to update daily volatilities.

5.6 Smoothing technique

The estimated volatilities contain a random variation component. This conceals some of the more useful components of the forecast hence the need for smoothing. Several options were considered for this purpose. These are the moving averages and the exponential smoothing.

5.6.1 Moving averages

This is computed as the arithmetic average for the time period with this period arbitrarily assigned. Thus, if a low volatility of the calculated ones (volatilities) is expected, this period would typically be short and the opposite would necessitate a longer period. Further, the first and last values of the data set are always lost if a two-day moving average is taken and the situation is worse if the period is increased. Perhaps one of the downfalls of this technique is the fact that historical data is systematically lost since only the data in the moving period is recognized in determining the smoothed volatility. The primary aim of this study is to test the accuracy of the different models relative to each
other and the implied volatility. The problem of lost data as explained earlier is shown in the graph below (the graph shows a generic situation).

Fig 5.1: Simulated moving average smoothing technique

The lag shown by the moving average means that statistical comparison of the volatility against a benchmark cannot be undertaken for the entire period. This method of smoothing was therefore not adopted for use in the study.

### 5.6.2 Exponential smoothing

This technique was used to recognize both the time series volatility and the smoothed volatility. In addition, historical movements are recognized since the immediate past smoothed volatility impound the day before it. The weight of the historical trends is thus recognized albeit less and less as time passes. The following formula was used in the smoothing process:

\[ S_t = wy_t + (1 - w)S_{t-1} \]

where
- \( S_t \) – exponentially smoothed time series at time \( t \)
- \( y_t \) – time series at time \( t \)
- \( S_{t-1} \) – smoothed time series at time \( t-1 \)
- \( w \) – smoothing constant and \( 0 < w < 1 \)
6. Findings

6.1 The implied volatility

The Black-Scholes implied volatility is dependent on the equation holding for a certain market price of the option and volatility. The price of the underlying asset is also an important element of this equation. There is thus a link between the price of the warrant and that of the underlying asset, albeit non-linear. When there is a price shock on the underlying asset, it is expected that the price of the warrant will reflect this. This was the case in most of the Warrants investigated. However, for MCELL, BARLOWS, JOHNNIC and STANBIC there were on several occasions sudden movements in the share price that were not reflected by the expected change in the warrant price. This led to a situation where mathematically, the implied volatility could be calculated as almost zero or negative. If the hypothesis that the implied volatility is the market’s view of the average volatility to maturity (or future volatility) is accepted, then an order of zero implied volatility is highly unlikely. The sudden movement in the price indicates a level of uncertainty in the market, essentially a source of volatility. There is therefore a contradiction between the effect of the shock and the theoretical explanation (as given by the Black-Scholes equation). The volatilities so obtained thus cannot be used as a benchmark for the forecasted ones.

The sector is very thinly traded. In fact, there are days where no trade was recorded for a particular Warrant. It is possible that some of the investors purchase Warrants not for trading but as long term investments. In this case, the market free float is extremely low hence the thin trade. The semi-strong form efficiency hypothesis is violated through the price not reflecting all of the publicly known information. Identification of these gaps could in fact lead to arbitrage opportunities for investors and therefore leave the warrant issuers vulnerable.

6.2 Performance of the volatility measures

The three measures were poor predictors of the implied volatility level. To get a global sense of the mis-specification, the residuals were expressed as a fraction of the actual implied volatility. The average error for a company was then obtained over the period of
the warrant under consideration. The average errors from the different companies were then tallied to get averages for the entire study. Appendix D gives a summary of this table.

The EGARCH method was on average 2.3% under specifying for the 20 trading day volatility forecast, with its smoothed version marginally better at 2.2%. This might at first glance imply a high level of accuracy. Taking the arithmetic average means that for a mean reverting time series, there is likelihood in time of high positive and negative terms cancelling out. This was the case in this instance and for some shares the over (under) specification was of the order of 50%.

The GARCH method was similar to the EGARCH. Here, the under specification was 4.8% and 4.5% for the GARCH and smoothed version respectively. Similarly, this cannot be said to be a good result on its own nor can it be said to be worse that the EGARCH method. The level of volatility of some of the forecasted volatilities meant that there were extensive fluctuations. Again, the cancellation effect led to the low average specification.

The EWMA which was used only to forecast daily volatilities returned different results to the above. Here, the average over specification was found to be 23.8% and 24.5% for the smoothed version. Unlike the other two methods, the smoothed version produced marginally worse results.

None of the measures mentioned above could on the face of the evidence produced be used as a trading tool for equity Warrants on the JSE. This is further worsened by the failure of the Black-Scholes model to produce a benchmark for all conditions. The STANBIC share for example was found to be over 82% under-specified by the EGARCH method. This was however off a base of almost zero since a feasible implied volatility could not be found.
6.3 Static convergence

For this, the correlations (R) were calculated between the implied volatility and the respective forecasts of the different measures. These could then be converted by squaring to get the coefficient of determination, $R^2$. With the high level of variability in the forecasted volatility, the correlations were found to be generally very small i.e. ranging from absolute 0.4 down to almost zero. To demonstrate the effect of the variability, IXCONE had an R value of 0.17 for the EWMA measure. By reducing the variability through smoothing, this was increased to 0.24 for the exponentially smoothed EWMA. The high variability thus reduces the importance of this as a measure of forecast accuracy. Further, no visible pattern of one method outperforming the others was found.

6.4 Sources of error

Two approaches can be taken with respect to the results presented thus far. One could be a conclusion that the models are inadequate for the purposes they are being used. The other is to look at other factors that could have affected the results. Two factors that will be considered are the problems with the maximum likelihood function (L) and the volatility smiles.

6.4.1 The maximum likelihood function

The weights in both auto-regressive models are determined using this function. Considering fig 6.1 below, L is plotted against the number of iterations to a global maximum.

![Maximum likelihood function](image.png)

Fig. 6.1: Maximum likelihood function
The optimisation algorithms need to be able to distinguish a global maximum C from local maxima A and B. The general purpose algorithm used for the study, Solver, seemed in some instances unable to find a global maximum. This was more so for the EGARCH model where running the algorithm repeatedly on the same data set gave different maxima, a sign of instability. This was partly overcome by running it (the algorithm) several times to get a sense of the order of magnitude of the maximum. The value of L so obtained is therefore not necessarily the global maximum but the best estimate.

The long run average variance is thus compromised by the lack of accuracy in the weights. Since the forecasting forms of the auto-regressive models have this average variance as an essential component, the level of accuracy of the forecasted volatilities is compromised. To overcome this problem, it is essential for future studies that a more specialised algorithm is utilized. Alternatively, these forecasts can be made using software packages like Eviews.

6.4.2 Volatility smiles

Using the Black-Scholes model, options valued as European with the same expiration date on the same underlying asset should have the same implied volatility. If this is violated, a neutral bull or bear spread trading strategy will lock in a profit even though both estimated volatilities are different from the true market volatility. Although this phnomena was not the subject of this study, table 6.1 is used is used to give a sense of the situation on the JSE (data source: SCMB).

For the first two shares (DIDATA and SAB), it expected that the implied volatility be the same. Clearly this is not the case, especially with SAB. On the day this data was taken (18 Feb 00), the warrant 3SAB was trading closer to the strike price than 1SAB. Consistent with the literature on volatility smiles, the implied volatility for the closer to the money option was the lower of the two. The DIDATA case can be viewed as a rounding off error.

There is also a discrepancy between the American Call and European Put for the same underlying share. For this study, it was assumed based on Merton (1973) that the
American Call options could be valued as European calls since they are likely to be held to maturity. If this is a correct assumption for the market conditions on the JSE, the two implied volatilities are then comparable. Again, there is a difference in the implied volatilities for ABSA and DEBEERS.

<table>
<thead>
<tr>
<th>Underlying Asset-Code</th>
<th>Expiry</th>
<th>Implied Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIDATA – 1DDT (AC)</td>
<td>15 June 01</td>
<td>43%</td>
</tr>
<tr>
<td>- 2DDT (AC)</td>
<td>15 June 01</td>
<td>42%</td>
</tr>
<tr>
<td>SAB – 1SAB (AC)</td>
<td>16 March 00</td>
<td>70%</td>
</tr>
<tr>
<td>- 3SAB (AC)</td>
<td>16 March 00</td>
<td>62%</td>
</tr>
<tr>
<td>ABSA – 2ASASB (AC)</td>
<td>15 March 01</td>
<td>88%</td>
</tr>
<tr>
<td>- 4ASASB (EP)</td>
<td>15 March 01</td>
<td>88%</td>
</tr>
<tr>
<td>ANGLO – 3AGL (AC)</td>
<td>16 March 00</td>
<td>44%</td>
</tr>
<tr>
<td>- 4AGL (EP)</td>
<td>16 March 00</td>
<td>59%</td>
</tr>
<tr>
<td>DEBEERS – 3DBR (AC)</td>
<td>16 March 00</td>
<td>67%</td>
</tr>
<tr>
<td>- 4DBR (EP)</td>
<td>16 March 00</td>
<td>41%</td>
</tr>
</tbody>
</table>

Table 6.1: Implied volatility estimates  (source: SCMB Financial markets handbook 2000)

Here there are two possibilities for this (difference), one being the fact that the market does not use the European Call market price as a proxy for that of the American Call. Alternatively, this might suggest some existence of smiles in the market although this cannot be concluded with any degree of certainty. However, another eight instances not given in table 6.1 were identified with a similar European Put / American Call discrepancy in implied volatility.

Clearly an error is introduced in the study regardless of the explanation for the European Put / American Call volatility discrepancy in table 6.1. A flat implied volatility smile allows for Black-Scholes pricing and therefore the corresponding volatility. Alternatively, the Black-Scholes offers a closed form solution and for a single set of data input will give a unique output. Thus, implied volatility smiles reduce the ability of the Black-Scholes model to give an accurate answer.
The relative performances of the different volatility measures with respect to accuracy can thus not be established. In fact, based on the findings, the measures can at best be used as a guide of likely market movements and determinants of level.

6.5 Dynamic convergence

Up to this point, the discussion has been based on a window period of observations, in a sense a ‘snapshot’ in time. It is not inconceivable that changing this time frame could lead to similar or markedly different results. To quantify this, a measure for the cointegration of the forecasted volatilities with the Black-Scholes implied volatilities was used. The Augmented Dickey Fuller test was used in this regard.

6.5.1 Test assumptions

The variables are tested for the presence of a unit root to determine whether or not they are stationary. To test for cointegration, a linear combination of the variables under test was tested for the presence of unit roots. In respect of this study, the residual as determined by the difference between the implied and the forecasted volatility, was used as this linear combination. Where the residual was found to be stationary i.e. I(0), the Durbin-Watson statistic was assessed for auto-correlation. Thus, the absence of auto-correlation and unit roots led to the conclusion of cointegration.

6.5.2 Results of the test

Referring to appendix E, the Augmented Dicky-Fuller test was failed at the 5% significance on two occasions (10% failure rate) by the EGARCH model (SASOL and SAPPI) and once for the GARCH model (SAPPI). For the EWMA, there was no failure. The Durbin Watson statistic generally did not exhibit a deviation towards auto-correlation for the residual. This seems to suggest that the residual (as determined by the three methods) generally exhibits mean reversion and has a finite variance that is time invariant.
By implication, the three methods are able to follow the implied volatility term structure in time. This could possibly explain the observation of the low average over (under) specification of the volatility as discussed earlier.

To assess the performance of one measure relative to the others, the level of the Augmented Dicky-Fuller static was used. Although the critical statistics were not exactly the same (due to different sample sizes), it was the same to two decimal points for all of the residuals tested. An assumption was thus made that the critical statistics were similar enough as not to introduce an error in the assessment as mentioned above. For the 21 tests performed the GARCH method proved superior 52% of the time, the EWMA 32% and the EGARCH 16%. Added to the fact that only one GARCH derived residual failed the Augmented Dicky-Fuller test, it would seem that this method offers superior trending abilities with the EGARCH the worst.
7. Conclusion

All the comparisons undertaken in the study are based on the assumption of the Black-Scholes option's ability to reflect the market's view on pricing and thus volatility. In addition, the assumption used to value the American call Warrants as European does not necessarily reflect the market view. There is therefore a potential for an error in the benchmark used, a factor that has the ability to give a false sense of performance for the different models. Brookes et al (1999) found that the Black-Scholes model could not explain the equity warrant prices. These findings could also have been turned around to the extent that the Brookes study did not use the correct values of volatility. There seems therefore to be a need for using other methods to calculate the implied volatilities of the market. This could then be used as an input to the Black-Scholes model and its efficacy determined. If it is found at that stage that the model is able to predict prices, then the use of Black-Scholes implied volatilities is justifiable and the results obtained in this study, notwithstanding the limitations already given in the report, give a fair sense of the relative performances of the models.

Most of the Warrants investigated had the underlying returns not normally distributed as tested using the Jarque-Bera statistic. Four of the twenty-one were found to be normally distributed at the 5% significance level and these are: SAB, ABSA, BARLOWS and FIRSTRAND. Since the Black-Scholes model assumes lognormality, there is likely to be an error in its use if this (lognormality) does not hold. From the study, there was no significant difference between the results from the four companies and the average results of the study; nor was there a discernible trend. It was thus concluded that the lack of normality of the data did not significantly affect the study.

Although none of the methods returned clearly superior results, the EWMA was on average the poorest point estimator. Because of its short-term nature, this (ability to provide point estimation) is an important quality. It is thus concluded that for the market under investigation, the EWMA cannot be used as a strategic investment tool. On the other hand the two auto-regressive techniques can be useful in predicting the future average volatility movements. As a tool for longer-term strategic investment, they may well be useful. Finally, the null hypothesis of the three methods’ ability to predict the term structure of implied volatility is rejected at the 5% significance level.
This study has highlighted several areas for future study. These are:

- **Implied volatilities:** Different techniques need to be investigated for calculating this. Because the market is thinly traded, there is a tendency for a de-linking of the underlying asset price and the warrant price. This leads to an infeasible Black-Scholes implied volatility.

- **Pricing of Warrants:** There is also a need for further investigation into an appropriate pricing model for equity Warrants in a thinly traded market. This study would have to further look at the markets tendency with regards to exercising American call Warrants before expiry. A discovery of a predominant trend to exercise at expiry would mean that these could be valued as European options. This would confirm the assumption made both by this study and **Brookes et al (1998)**.
8. Bibliography


Brooke G et al 1999,“Valuing JSE listed Warrants using the Black-Scholes option pricing model”, Unpublished draft, University of Cape Town.


APPENDIX A:

A.1 Determination of daily returns

For the purposes of the study, it is assumed that the returns are continuously compounded. Thus, to calculate the return today on yesterday’s price, this is simply,

\[ S_i e^{r_i} = S_{i+1} \]

where \( r \) is the daily return. However, this form assumes returns over one day only. Part of the data passes over public holidays and weekends. If the above form is still used, it is will overstate the returns on days immediately following non-trading days. Thus, a more general form is,

\[ S_i e^{r_m} = S_{i+m} \]

where \( m \) is the number of periods over which the returns are compounded. Rearranging for the return, \( r \),

\[ r = \frac{1}{m} \ln \left( \frac{S_{i+m}}{S_i} \right) \]

A.2 Forecasting share prices

If a view is taken on the future volatility and this could be done with good accuracy, the corresponding share price for the day in question can be analytically determined. In reality however, the volatility is itself very volatile and thus this technique is liable to produce inaccurate results. Although not used in this study, it is presented here.
For day $i$,

$$S_i = S_{i-1} \exp(\mu + \varepsilon \sigma_i)$$

With day $i-n$ expressed as,

$$S_{i-n} = S_{i-1-n} \exp(\mu + \varepsilon \sigma_{i-n})$$

Substituting backwards, the share price for day $i$ is then expressed in terms of today's ($i-n$) price as,

$$S_i = S_{i-n} \exp\left(\sum_{i-n}^{i} (\mu + \varepsilon \sigma_i)\right)$$

where the volatilities to day $i$ have been estimated.
APPENDIX B: Normality tests for daily returns

<table>
<thead>
<tr>
<th>Company</th>
<th>VALID_N</th>
<th>MEAN</th>
<th>MINIMUM</th>
<th>MAXIMUM</th>
<th>SKEWNESS</th>
<th>JARQUE-BERA</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANGLO</td>
<td>743</td>
<td>-0.18232</td>
<td>0.140507</td>
<td>-0.295771</td>
<td>4.657512</td>
<td>95.8863547</td>
</tr>
<tr>
<td>BIDVEST</td>
<td>743</td>
<td>-0.10536</td>
<td>0.123845</td>
<td>-0.067681</td>
<td>2.331885</td>
<td>14.3864623</td>
</tr>
<tr>
<td>COMPAREX</td>
<td>743</td>
<td>-1.13873</td>
<td>0.138231</td>
<td>-12.63906</td>
<td>3.997568</td>
<td>2116213.36</td>
</tr>
<tr>
<td>DEBEERS</td>
<td>743</td>
<td>-0.1151</td>
<td>0.106413</td>
<td>-0.260212</td>
<td>2.737237</td>
<td>20.5448176</td>
</tr>
<tr>
<td>GENCOR</td>
<td>743</td>
<td>-0.50078</td>
<td>0.154151</td>
<td>-3.48688</td>
<td>51.211485</td>
<td>73463.5263</td>
</tr>
<tr>
<td>HARMONY</td>
<td>743</td>
<td>-0.15587</td>
<td>0.228684</td>
<td>0.776559</td>
<td>3.997568</td>
<td>105.484906</td>
</tr>
<tr>
<td>INVESTEC</td>
<td>743</td>
<td>-0.14266</td>
<td>0.099274</td>
<td>-0.505514</td>
<td>5.807005</td>
<td>275.574314</td>
</tr>
<tr>
<td>JOHNNIC</td>
<td>743</td>
<td>-0.20252</td>
<td>0.217413</td>
<td>-0.027943</td>
<td>4.454104</td>
<td>65.555612</td>
</tr>
<tr>
<td>METCASH</td>
<td>743</td>
<td>-0.15155</td>
<td>0.182322</td>
<td>0.1914592</td>
<td>3.297687</td>
<td>7.41170397</td>
</tr>
<tr>
<td>PROFURN</td>
<td>743</td>
<td>-0.14764</td>
<td>0.143707</td>
<td>-0.105602</td>
<td>2.791897</td>
<td>2.72166754</td>
</tr>
<tr>
<td>RICHEMON</td>
<td>743</td>
<td>-0.12736</td>
<td>0.112182</td>
<td>0.1821633</td>
<td>4.749747</td>
<td>98.8917066</td>
</tr>
<tr>
<td>SAPPI</td>
<td>743</td>
<td>-0.19268</td>
<td>0.213193</td>
<td>0.442706</td>
<td>5.185911</td>
<td>172.682966</td>
</tr>
<tr>
<td>STANBIC</td>
<td>743</td>
<td>-0.18628</td>
<td>0.13787</td>
<td>-0.485506</td>
<td>4.255358</td>
<td>78.3394085</td>
</tr>
<tr>
<td>AMPLATS</td>
<td>743</td>
<td>-0.16632</td>
<td>0.147093</td>
<td>0.213499</td>
<td>4.753717</td>
<td>100.857726</td>
</tr>
<tr>
<td>ANGLOGOL</td>
<td>743</td>
<td>-0.09531</td>
<td>0.223245</td>
<td>1.141929</td>
<td>5.804718</td>
<td>405.010927</td>
</tr>
<tr>
<td>BILLITON</td>
<td>743</td>
<td>-0.16127</td>
<td>0.141798</td>
<td>0.2416548</td>
<td>3.755825</td>
<td>24.9171446</td>
</tr>
<tr>
<td>CORONATI</td>
<td>743</td>
<td>-0.18232</td>
<td>0.160581</td>
<td>-0.442553</td>
<td>7.515650</td>
<td>655.527551</td>
</tr>
<tr>
<td>DIDATA</td>
<td>743</td>
<td>-0.15719</td>
<td>0.164229</td>
<td>-0.101203</td>
<td>4.195926</td>
<td>44.2933691</td>
</tr>
<tr>
<td>GENSEC</td>
<td>743</td>
<td>-0.17196</td>
<td>0.248833</td>
<td>0.0736305</td>
<td>6.752456</td>
<td>235.212281</td>
</tr>
<tr>
<td>IMPERIAL</td>
<td>743</td>
<td>-0.16617</td>
<td>0.123614</td>
<td>-0.204144</td>
<td>4.828473</td>
<td>106.646123</td>
</tr>
<tr>
<td>ISCOR</td>
<td>743</td>
<td>-0.16632</td>
<td>0.147093</td>
<td>0.213499</td>
<td>4.753717</td>
<td>100.857726</td>
</tr>
<tr>
<td>LIBERTY</td>
<td>743</td>
<td>-0.43383</td>
<td>0.158969</td>
<td>-3.740317</td>
<td>47.007397</td>
<td>61687.9128</td>
</tr>
<tr>
<td>NEDCOR</td>
<td>743</td>
<td>-0.03749</td>
<td>0.155149</td>
<td>0.034221</td>
<td>3.691982</td>
<td>14.9690817</td>
</tr>
<tr>
<td>REBOLD</td>
<td>743</td>
<td>-0.15954</td>
<td>0.177887</td>
<td>-0.112545</td>
<td>5.155250</td>
<td>145.3732</td>
</tr>
<tr>
<td>SAB</td>
<td>743</td>
<td>-0.13676</td>
<td>0.132976</td>
<td>-0.073347</td>
<td>3.027873</td>
<td>690.2514</td>
</tr>
<tr>
<td>SASOL</td>
<td>743</td>
<td>-0.17355</td>
<td>0.142908</td>
<td>-0.154864</td>
<td>3.542129</td>
<td>12.067746</td>
</tr>
<tr>
<td>TONGAAT</td>
<td>743</td>
<td>-0.2041</td>
<td>0.126752</td>
<td>-0.193937</td>
<td>3.462134</td>
<td>11.2693145</td>
</tr>
<tr>
<td>ABSA</td>
<td>743</td>
<td>-0.17042</td>
<td>0.141332</td>
<td>-0.150682</td>
<td>3.084406</td>
<td>3.03218225</td>
</tr>
<tr>
<td>BARLOWS</td>
<td>743</td>
<td>-0.14266</td>
<td>0.102072</td>
<td>-0.19772</td>
<td>2.801278</td>
<td>6.06361331</td>
</tr>
<tr>
<td>BOE</td>
<td>743</td>
<td>-0.37469</td>
<td>0.15104</td>
<td>-1.693297</td>
<td>17.96612</td>
<td>7289.25586</td>
</tr>
<tr>
<td>DATATEC</td>
<td>743</td>
<td>-0.38829</td>
<td>0.178692</td>
<td>-2.481029</td>
<td>20.245674</td>
<td>9969.67606</td>
</tr>
<tr>
<td>FIRSTRAN</td>
<td>743</td>
<td>-0.11709</td>
<td>0.138567</td>
<td>0.2679768</td>
<td>2.920088</td>
<td>9.09036429</td>
</tr>
<tr>
<td>GOLDFIEL</td>
<td>743</td>
<td>-0.13528</td>
<td>0.249035</td>
<td>0.9792452</td>
<td>5.863497</td>
<td>372.5862</td>
</tr>
<tr>
<td>IMPLATS</td>
<td>743</td>
<td>-0.20585</td>
<td>0.149036</td>
<td>-0.005831</td>
<td>5.316021</td>
<td>166.063389</td>
</tr>
<tr>
<td>IXCHANGE</td>
<td>743</td>
<td>-0.38677</td>
<td>0.294239</td>
<td>0.0110678</td>
<td>9.5961975</td>
<td>1347.00674</td>
</tr>
<tr>
<td>MCELL</td>
<td>743</td>
<td>-0.02122</td>
<td>0.20764</td>
<td>0.217113</td>
<td>-0.003894</td>
<td>5.1883216</td>
</tr>
<tr>
<td>OLDUMUTUA</td>
<td>292</td>
<td>-0.07889</td>
<td>0.152244</td>
<td>1.0130312</td>
<td>6.6918959</td>
<td>215.776129</td>
</tr>
<tr>
<td>REMRANDT</td>
<td>743</td>
<td>-0.16629</td>
<td>0.112293</td>
<td>-0.493519</td>
<td>5.0275922</td>
<td>157.434648</td>
</tr>
<tr>
<td>SANLAM</td>
<td>443</td>
<td>-0.09867</td>
<td>0.131028</td>
<td>0.6297883</td>
<td>3.509551</td>
<td>34.0773199</td>
</tr>
<tr>
<td>SOFTLINE</td>
<td>743</td>
<td>-0.24946</td>
<td>0.202941</td>
<td>-0.129518</td>
<td>4.457645</td>
<td>67.8554107</td>
</tr>
<tr>
<td>WOOLTR_N</td>
<td>743</td>
<td>-0.01355</td>
<td>-0.39054</td>
<td>0.121607</td>
<td>-2.762963</td>
<td>32.042294</td>
</tr>
</tbody>
</table>
APPENDIX C: Computer code used

C.1 Implied Volatility

Sub v2vanglo()

Dim j As Integer
Dim sheetsglobal As Worksheet
Dim sheetslocal As Sheets
Dim workbookglobal As Workbook

Set sheetslocal = ThisWorkbook.Sheets

For Each sheetsglobal In sheetslocal
    sheetsglobal.Activate

    For j = 1 To Range("K2:K600").Rows.Count Step 1
        SolverReset
        SolverOptions Precision:=0.001
        SolverOk SetCell:=Range("K2:K600").Rows(j), MaxMinVal:=3, Valueof:=Range("D2:D600").Rows(j).Value, ByChange:=Range("E2:E600").Rows(j)
        SolverFinish KeepFinal:=1
        SolverSolve userFinish:=True

    Next

    Next sheetsglobal

End Sub
C.2 GARCH and EGARCH Maximum likelihood calculation

Sub maxlikelihood ()

Dim sheetsglobal1 As Worksheet
Dim sheetslocal1 As Sheets
Dim workbookglobal1 As Workbook

Set sheetslocal1 = ThisWorkbook.Sheets

For Each sheetsglobal1 In sheetslocal1
    sheetsglobal1.Activate

% Solving for EGARCH

SolverReset
SolverOptions Precision:=0.001
SolverAdd CellRef:=Range("S5"), Relation:=3, FormulaText:=0.0004
SolverAdd CellRef:=Range("S3"), Relation:=1, FormulaText:=0.9999999999

SolverOk SetCell:=Range("S2"), MaxMinVal:=1, ByChange:=Range("Q1:Q4")
SolverFinish KeepFinal:=1
SolverSolve userFinish:=True

% Solving for GARCH

SolverReset
SolverOptions Precision:=0.001
SolverAdd CellRef:=Range("G4"), Relation:=3, FormulaText:=0.0000000001
SolverAdd CellRef:=Range("G5"), Relation:=1, FormulaText:=0.9999999999
SolverAdd CellRef:=Range("H4"), Relation:=1, FormulaText:=0.9999999999
SolverOk SetCell:=Range("G6"), MaxMinVal:=1, ByChange:=Range("G1:G3")
SolverFinish KeepFinal:=1
SolverSolve userFinish:=True

Next sheetsglobal1

End Sub
**APPENDIX D:**

**D.1 RESIDUAL ERRORS**

<table>
<thead>
<tr>
<th>Company name</th>
<th>Code</th>
<th>EG</th>
<th>EGES</th>
<th>G</th>
<th>GES</th>
<th>EW</th>
<th>EWES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firststrand</td>
<td>FSR</td>
<td>0.077881081</td>
<td>0.075994</td>
<td>0.147556</td>
<td>0.144821</td>
<td>0.411273</td>
<td>0.408266</td>
</tr>
<tr>
<td>STANBIC</td>
<td>SBC</td>
<td>-0.82181179</td>
<td>-0.79893</td>
<td>-0.78192</td>
<td>-0.85991</td>
<td>-0.1889</td>
<td>-0.19915</td>
</tr>
<tr>
<td>SAPPi</td>
<td>SAPDB</td>
<td>0.431317003</td>
<td>0.431149</td>
<td>0.517398</td>
<td>0.517394</td>
<td>0.625307</td>
<td>0.624473</td>
</tr>
<tr>
<td>Remgro</td>
<td>RMT</td>
<td>0.40399811</td>
<td>0.405147</td>
<td>0.514151</td>
<td>0.513986</td>
<td>0.621408</td>
<td>0.618063</td>
</tr>
<tr>
<td>IXCHANGE</td>
<td>XCHSG</td>
<td>0.35475942</td>
<td>0.352965</td>
<td>0.386034</td>
<td>0.38607</td>
<td>0.472366</td>
<td>0.474244</td>
</tr>
<tr>
<td>Tongaat</td>
<td>TNTSG</td>
<td>0.174666565</td>
<td>0.176316</td>
<td>0.21161</td>
<td>0.202362</td>
<td>0.386721</td>
<td>0.384885</td>
</tr>
<tr>
<td>SAB</td>
<td>SAB</td>
<td>0.18641738</td>
<td>0.176316</td>
<td>0.271326</td>
<td>0.269287</td>
<td>0.446391</td>
<td>0.431029</td>
</tr>
<tr>
<td>SASOL</td>
<td>SOLSB</td>
<td>0.46932063</td>
<td>0.469788</td>
<td>0.461084</td>
<td>0.460179</td>
<td>0.617791</td>
<td>0.622425</td>
</tr>
<tr>
<td>BILLITON</td>
<td>BIL</td>
<td>-0.32088032</td>
<td>-0.27557</td>
<td>-0.21108</td>
<td>-0.19667</td>
<td>-0.28242</td>
<td>-0.13689</td>
</tr>
<tr>
<td>AMPLATS</td>
<td>AMSD</td>
<td>0.186700686</td>
<td>0.18757</td>
<td>0.247021</td>
<td>0.251024</td>
<td>0.388503</td>
<td>0.390104</td>
</tr>
<tr>
<td>Nedcor</td>
<td>NED</td>
<td>-0.00995895</td>
<td>-0.01263</td>
<td>0.041005</td>
<td>0.040523</td>
<td>0.24862</td>
<td>0.246496</td>
</tr>
<tr>
<td>MCELL</td>
<td>MCE</td>
<td>-1.02095067</td>
<td>-1.09298</td>
<td>-1.1192</td>
<td>-1.10677</td>
<td>-1.07407</td>
<td>-1.161</td>
</tr>
<tr>
<td>IMPERIAL</td>
<td>IPL</td>
<td>-0.34214151</td>
<td>-0.33617</td>
<td>-0.22397</td>
<td>-0.22431</td>
<td>0.308221</td>
<td>0.382125</td>
</tr>
<tr>
<td>JOHNNIC</td>
<td>JNC</td>
<td>-0.95289855</td>
<td>-1.00163</td>
<td>-1.11679</td>
<td>-1.10727</td>
<td>-1.01494</td>
<td>-1.0916</td>
</tr>
<tr>
<td>ABSA</td>
<td>ASA</td>
<td>0.0466844</td>
<td>-0.04719</td>
<td>0.066493</td>
<td>0.066214</td>
<td>0.340905</td>
<td>0.336276</td>
</tr>
<tr>
<td>HARMONY</td>
<td>HARS</td>
<td>0.03720972</td>
<td>0.036335</td>
<td>0.204181</td>
<td>0.202266</td>
<td>0.40525</td>
<td>0.401402</td>
</tr>
<tr>
<td>ANGLO</td>
<td>AGL</td>
<td>-0.219605</td>
<td>-0.22155</td>
<td>-0.01954</td>
<td>-0.01674</td>
<td>0.193974</td>
<td>0.199339</td>
</tr>
<tr>
<td>DEBEERS</td>
<td>DBR</td>
<td>0.02830622</td>
<td>-0.02116</td>
<td>0.05796</td>
<td>0.060938</td>
<td>0.193422</td>
<td>0.193793</td>
</tr>
<tr>
<td>IMPLATS</td>
<td>IMP</td>
<td>-0.10654206</td>
<td>-0.11148</td>
<td>-0.03241</td>
<td>-0.02864</td>
<td>0.094984</td>
<td>0.124359</td>
</tr>
<tr>
<td>DIDATA</td>
<td>DD</td>
<td>0.048044496</td>
<td>0.048244</td>
<td>0.179334</td>
<td>0.179927</td>
<td>0.257765</td>
<td>0.25948</td>
</tr>
<tr>
<td>BARLOWS</td>
<td>BAR</td>
<td>-2.6451948</td>
<td>-2.52223</td>
<td>-2.58755</td>
<td>-2.45783</td>
<td>-1.62406</td>
<td>-1.74563</td>
</tr>
</tbody>
</table>

**Average**

-0.02273518 -0.022050 0.047866 0.045339 0.238244 0.245743

(excl BARLOWS and MCELL)
### D.2 CORRELATIONS

<table>
<thead>
<tr>
<th>FirstRand</th>
<th>(\text{Implied Volatility})</th>
<th>SASOL</th>
<th>(\text{Implied Volatility})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Implied Volatility</td>
<td>1</td>
<td>Implied Volatility</td>
<td>1</td>
</tr>
<tr>
<td>EGarch</td>
<td>0.135896545</td>
<td>EGarch</td>
<td>0.037210853</td>
</tr>
<tr>
<td>Exp Smooth</td>
<td>0.199731059</td>
<td>Exp Smooth</td>
<td>0.029085851</td>
</tr>
<tr>
<td>Garch</td>
<td>-0.001313741</td>
<td>Garch</td>
<td>0.052463779</td>
</tr>
<tr>
<td>Exp Smooth</td>
<td>0.100727265</td>
<td>Exp Smooth</td>
<td>0.044530877</td>
</tr>
<tr>
<td>EWMA</td>
<td>-0.134414706</td>
<td>EWMA</td>
<td>0.323974701</td>
</tr>
<tr>
<td>Exp Smooth</td>
<td>-0.173439043</td>
<td>Exp Smooth</td>
<td>0.402528177</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>STANBIC</th>
<th>(\text{Implied Volatility})</th>
<th>BILLITON</th>
<th>(\text{Implied Volatility})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Implied Volatility</td>
<td>1</td>
<td>Implied Volatility</td>
<td>1</td>
</tr>
<tr>
<td>EGarch</td>
<td>0.074569182</td>
<td>EGarch</td>
<td>0.157550672</td>
</tr>
<tr>
<td>Exp Smooth</td>
<td>0.108230113</td>
<td>Exp Smooth</td>
<td>0.251433269</td>
</tr>
<tr>
<td>Garch</td>
<td>0.063288653</td>
<td>Garch</td>
<td>0.05598921</td>
</tr>
<tr>
<td>Exp Smooth</td>
<td>0.096908249</td>
<td>Exp Smooth</td>
<td>0.164707593</td>
</tr>
<tr>
<td>EWMA</td>
<td>-0.014881838</td>
<td>EWMA</td>
<td>0.003962268</td>
</tr>
<tr>
<td>Exp Smooth</td>
<td>0.018572459</td>
<td>Exp Smooth</td>
<td>0.06188478</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SAPPI</th>
<th>(\text{Implied Volatility})</th>
<th>AMPLATS</th>
<th>(\text{Implied Volatility})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Implied Volatility</td>
<td>1</td>
<td>Implied Volatility</td>
<td>1</td>
</tr>
<tr>
<td>EGarch</td>
<td>-0.121062137</td>
<td>EGarch</td>
<td>-0.016897967</td>
</tr>
<tr>
<td>Exp Smooth</td>
<td>-0.169763024</td>
<td>Exp Smooth</td>
<td>-0.045866704</td>
</tr>
<tr>
<td>Garch</td>
<td>0.00396497</td>
<td>Garch</td>
<td>-0.07813636</td>
</tr>
<tr>
<td>Exp Smooth</td>
<td>0.009765059</td>
<td>Exp Smooth</td>
<td>-0.10645984</td>
</tr>
<tr>
<td>EWMA</td>
<td>0.025156027</td>
<td>EWMA</td>
<td>0.136243673</td>
</tr>
<tr>
<td>Exp Smooth</td>
<td>-0.00894772</td>
<td>Exp Smooth</td>
<td>0.204451415</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>REMGRO</th>
<th>(\text{Implied Volatility})</th>
<th>NEDCOR</th>
<th>(\text{Implied Volatility})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Implied Volatility</td>
<td>1</td>
<td>Implied Volatility</td>
<td>1</td>
</tr>
<tr>
<td>EGarch</td>
<td>0.001746669</td>
<td>EGarch</td>
<td>0.101798541</td>
</tr>
<tr>
<td>Exp Smooth</td>
<td>0.032165489</td>
<td>Exp Smooth</td>
<td>0.141799138</td>
</tr>
<tr>
<td>Garch</td>
<td>0.141855184</td>
<td>Garch</td>
<td>0.078749664</td>
</tr>
<tr>
<td>Exp Smooth</td>
<td>0.152576857</td>
<td>Exp Smooth</td>
<td>0.108116516</td>
</tr>
<tr>
<td>EWMA</td>
<td>-0.012351952</td>
<td>EWMA</td>
<td>-0.001181852</td>
</tr>
<tr>
<td>Exp Smooth</td>
<td>-0.104917522</td>
<td>Exp Smooth</td>
<td>-0.053521186</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>IXCHANGE</th>
<th>(\text{Implied Volatility})</th>
<th>MCELL</th>
<th>(\text{Implied Volatility})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Implied Volatility</td>
<td>1</td>
<td>Implied Volatility</td>
<td>1</td>
</tr>
<tr>
<td>EGarch</td>
<td>0.080761541</td>
<td>EGarch</td>
<td>0.097269325</td>
</tr>
<tr>
<td>Exp Smooth</td>
<td>0.092108872</td>
<td>Exp Smooth</td>
<td>0.044611779</td>
</tr>
<tr>
<td>Garch</td>
<td>0.071532704</td>
<td>Garch</td>
<td>0.076181603</td>
</tr>
<tr>
<td>Exp Smooth</td>
<td>0.099000192</td>
<td>Exp Smooth</td>
<td>0.121492886</td>
</tr>
<tr>
<td>EWMA</td>
<td>0.170345211</td>
<td>EWMA</td>
<td>0.121891892</td>
</tr>
<tr>
<td>Exp Smooth</td>
<td>0.247100253</td>
<td>Exp Smooth</td>
<td>0.09328575</td>
</tr>
<tr>
<td>Company</td>
<td>Implied Volatility</td>
<td>EGarch</td>
<td>Exp Smoothing</td>
</tr>
<tr>
<td>---------</td>
<td>--------------------</td>
<td>--------</td>
<td>---------------</td>
</tr>
<tr>
<td><strong>TONGAAT</strong></td>
<td>Implied Volatility 1</td>
<td>-0.215233574</td>
<td>-0.200179159</td>
</tr>
<tr>
<td><strong>SAB</strong></td>
<td>Implied Volatility 1</td>
<td>0.107930122</td>
<td>0.160498499</td>
</tr>
<tr>
<td><strong>ABSA</strong></td>
<td>Implied Volatility 1</td>
<td>0.146972332</td>
<td>0.257871929</td>
</tr>
<tr>
<td><strong>HARMONY</strong></td>
<td>Implied Volatility 1</td>
<td>-0.024964248</td>
<td>-0.303856374</td>
</tr>
<tr>
<td><strong>ANGLO</strong></td>
<td>Implied Volatility 1</td>
<td>0.290623965</td>
<td>0.406426001</td>
</tr>
<tr>
<td><strong>BARLOWS</strong></td>
<td>Implied Volatility 1</td>
<td>-0.087059892</td>
<td>-0.101780756</td>
</tr>
<tr>
<td><strong>IMPERIAL</strong></td>
<td>Implied Volatility 1</td>
<td>0.083329974</td>
<td>0.050914258</td>
</tr>
<tr>
<td><strong>JOHNNIC</strong></td>
<td>Implied Volatility 1</td>
<td>-0.010008412</td>
<td>-0.069178964</td>
</tr>
<tr>
<td><strong>DEBEERS</strong></td>
<td>Implied Volatility 1</td>
<td>0.058241629</td>
<td>0.234149057</td>
</tr>
<tr>
<td><strong>IMPLATS</strong></td>
<td>Implied Volatility 1</td>
<td>0.105143473</td>
<td>0.043160539</td>
</tr>
<tr>
<td><strong>DIDATA</strong></td>
<td>Implied Volatility 1</td>
<td>0.165575455</td>
<td>0.27810819</td>
</tr>
<tr>
<td><strong>BARLOWS</strong></td>
<td>Implied Volatility 1</td>
<td>-0.087059892</td>
<td>-0.101780756</td>
</tr>
</tbody>
</table>
## APPENDIX E: DICKY FULLER TEST

<table>
<thead>
<tr>
<th>COMPANY</th>
<th>TEST</th>
<th>EG</th>
<th>EGES</th>
<th>G</th>
<th>GES</th>
<th>EW</th>
<th>EWES</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANGLO</td>
<td>ADF</td>
<td>-4.92625</td>
<td>-4.17927</td>
<td>-8.92131</td>
<td>-5.30716</td>
<td>-7.61662</td>
<td>-3.50219</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>1.705111</td>
<td>1.652931</td>
<td>1.988966</td>
<td>1.542668</td>
<td>2.030128</td>
<td>1.345291</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.9407</td>
<td>-1.9407</td>
<td>-1.9407</td>
<td>-1.9407</td>
<td>-1.9407</td>
<td>-1.9407</td>
</tr>
<tr>
<td>DEBEERS</td>
<td>ADF</td>
<td>-6.70522</td>
<td>-5.11072</td>
<td>-8.96458</td>
<td>-5.3446</td>
<td>-7.63614</td>
<td>-3.86078</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>1.62632</td>
<td>1.388384</td>
<td>1.96066</td>
<td>1.484827</td>
<td>1.968752</td>
<td>1.31366</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.9407</td>
<td>-1.9407</td>
<td>-1.9407</td>
<td>-1.9407</td>
<td>-1.9407</td>
<td>-1.9407</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>1.722443</td>
<td>1.726523</td>
<td>1.882914</td>
<td>1.711628</td>
<td>1.951858</td>
<td>1.688286</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.9403</td>
<td>-1.9403</td>
<td>-1.9403</td>
<td>-1.9403</td>
<td>-1.9403</td>
<td>-1.9403</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>2.044002</td>
<td>1.755923</td>
<td>2.174386</td>
<td>1.684212</td>
<td>2.112736</td>
<td>1.491958</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.94</td>
<td>-1.94</td>
<td>-1.94</td>
<td>-1.94</td>
<td>-1.94</td>
<td>-1.94</td>
</tr>
<tr>
<td>IMPLATS</td>
<td>ADF</td>
<td>-5.48139</td>
<td>-5.41375</td>
<td>-5.64064</td>
<td>-4.89677</td>
<td>-5.01102</td>
<td>-3.73849</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>1.617554</td>
<td>1.59115</td>
<td>1.87653</td>
<td>1.628701</td>
<td>1.876088</td>
<td>1.594343</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.9408</td>
<td>-1.9408</td>
<td>-1.9408</td>
<td>-1.9408</td>
<td>-1.9408</td>
<td>-1.9408</td>
</tr>
<tr>
<td>HARMONY</td>
<td>ADF</td>
<td>-4.47163</td>
<td>-4.20727</td>
<td>-5.37471</td>
<td>-3.60153</td>
<td>-4.16616</td>
<td>-2.48557</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>1.785291</td>
<td>1.702082</td>
<td>2.243429</td>
<td>1.933119</td>
<td>2.113492</td>
<td>1.440277</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.942</td>
<td>-1.942</td>
<td>-1.942</td>
<td>-1.942</td>
<td>-1.942</td>
<td>-1.942</td>
</tr>
<tr>
<td>ABSA</td>
<td>ADF</td>
<td>-6.93115</td>
<td>-3.89017</td>
<td>-8.62804</td>
<td>-4.06232</td>
<td>-6.3906</td>
<td>-2.45514</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>1.933814</td>
<td>1.435217</td>
<td>2.13917</td>
<td>1.564265</td>
<td>2.126404</td>
<td>1.428644</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.9403</td>
<td>-1.9403</td>
<td>-1.9403</td>
<td>-1.9403</td>
<td>-1.9403</td>
<td>-1.9403</td>
</tr>
<tr>
<td>JOHNNIC</td>
<td>ADF</td>
<td>-6.14032</td>
<td>-5.82242</td>
<td>-6.86996</td>
<td>-6.48413</td>
<td>-5.97661</td>
<td>-5.10889</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>2.097331</td>
<td>2.038512</td>
<td>1.998012</td>
<td>1.989066</td>
<td>1.925058</td>
<td>1.973509</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.9411</td>
<td>-1.9411</td>
<td>-1.9411</td>
<td>-1.9411</td>
<td>-1.9411</td>
<td>-1.9411</td>
</tr>
<tr>
<td>MCELL</td>
<td>ADF</td>
<td>-5.65496</td>
<td>-5.56727</td>
<td>-5.83228</td>
<td>-5.70792</td>
<td>-6.12475</td>
<td>-5.57941</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>1.739301</td>
<td>1.707279</td>
<td>1.742057</td>
<td>1.69876</td>
<td>1.663908</td>
<td>1.644533</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.9411</td>
<td>-1.9411</td>
<td>-1.9411</td>
<td>-1.9411</td>
<td>-1.9411</td>
<td>-1.9411</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>1.642675</td>
<td>1.292447</td>
<td>1.979869</td>
<td>1.347267</td>
<td>2.049603</td>
<td>1.23218</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.9403</td>
<td>-1.9403</td>
<td>-1.9403</td>
<td>-1.9403</td>
<td>-1.9403</td>
<td>-1.9403</td>
</tr>
<tr>
<td>AMPLATS</td>
<td>ADF</td>
<td>-4.09668</td>
<td>-4.24033</td>
<td>-4.18752</td>
<td>-2.23213</td>
<td>-4.65726</td>
<td>-2.23271</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>1.965946</td>
<td>1.760662</td>
<td>2.000723</td>
<td>1.76597</td>
<td>2.016142</td>
<td>1.591646</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.9414</td>
<td>-1.9414</td>
<td>-1.9414</td>
<td>-1.9414</td>
<td>-1.9414</td>
<td>-1.9414</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>1.763645</td>
<td>1.649174</td>
<td>1.975218</td>
<td>1.597673</td>
<td>2.031901</td>
<td>1.617829</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.941</td>
<td>-1.941</td>
<td>-1.941</td>
<td>-1.941</td>
<td>-1.941</td>
<td>-1.941</td>
</tr>
<tr>
<td>SASOL</td>
<td>ADF</td>
<td>-1.63446</td>
<td>-0.97695</td>
<td>-1.81396</td>
<td>-0.95632</td>
<td>-2.27163</td>
<td>-1.03333</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>2.311994</td>
<td>1.906132</td>
<td>2.470296</td>
<td>1.985551</td>
<td>2.402047</td>
<td>1.798756</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.9421</td>
<td>-1.9421</td>
<td>-1.9421</td>
<td>-1.9421</td>
<td>-1.9421</td>
<td>-1.9421</td>
</tr>
<tr>
<td>COMPANY</td>
<td>TEST</td>
<td>EG</td>
<td>EGES</td>
<td>G</td>
<td>GES</td>
<td>EW</td>
<td>EWES</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
<td>----</td>
<td>------</td>
<td>---</td>
<td>-----</td>
<td>----</td>
<td>------</td>
</tr>
<tr>
<td>SAB</td>
<td>ADF</td>
<td>-6.66445</td>
<td>-3.05139</td>
<td>-4.68114</td>
<td>-2.45885</td>
<td>-3.79509</td>
<td>-1.74679</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>1.948642</td>
<td>1.386915</td>
<td>2.278363</td>
<td>1.731611</td>
<td>2.153087</td>
<td>1.446061</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.9411</td>
<td>1.386915</td>
<td>2.278363</td>
<td>1.731611</td>
<td>2.153087</td>
<td>1.446061</td>
</tr>
<tr>
<td>TONGAAT</td>
<td>ADF</td>
<td>-2.40035</td>
<td>-1.41604</td>
<td>-4.85416</td>
<td>-1.82845</td>
<td>-4.18541</td>
<td>-1.47744</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>1.85469</td>
<td>1.368261</td>
<td>2.16962</td>
<td>1.544749</td>
<td>2.16684</td>
<td>1.527832</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.942</td>
<td>1.368261</td>
<td>2.16962</td>
<td>1.544749</td>
<td>2.16684</td>
<td>1.527832</td>
</tr>
<tr>
<td>IXCHANGE</td>
<td>ADF</td>
<td>-2.73647</td>
<td>-1.65535</td>
<td>-2.28115</td>
<td>-0.93286</td>
<td>-4.12713</td>
<td>-1.58201</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>2.039434</td>
<td>1.603044</td>
<td>2.150273</td>
<td>1.564421</td>
<td>2.206433</td>
<td>1.620381</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.942</td>
<td>1.603044</td>
<td>2.150273</td>
<td>1.564421</td>
<td>2.206433</td>
<td>1.620381</td>
</tr>
<tr>
<td>REMGRO</td>
<td>ADF</td>
<td>-3.05871</td>
<td>-1.99797</td>
<td>-2.56262</td>
<td>-1.57756</td>
<td>-2.97801</td>
<td>-1.46047</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>2.139596</td>
<td>1.845029</td>
<td>2.409284</td>
<td>2.030555</td>
<td>2.132724</td>
<td>1.499056</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.9405</td>
<td>1.845029</td>
<td>2.409284</td>
<td>2.030555</td>
<td>2.132724</td>
<td>1.499056</td>
</tr>
<tr>
<td>SAPP</td>
<td>ADF</td>
<td>-1.63049</td>
<td>-1.40344</td>
<td>-2.57374</td>
<td>-1.29586</td>
<td>-3.20614</td>
<td>-1.42763</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>1.985315</td>
<td>1.900592</td>
<td>2.475667</td>
<td>1.888114</td>
<td>2.182742</td>
<td>1.489855</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.9403</td>
<td>1.900592</td>
<td>2.475667</td>
<td>1.888114</td>
<td>2.182742</td>
<td>1.489855</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>1.845527</td>
<td>1.657879</td>
<td>1.955946</td>
<td>1.517213</td>
<td>1.993252</td>
<td>1.489855</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.9403</td>
<td>1.657879</td>
<td>1.955946</td>
<td>1.517213</td>
<td>1.993252</td>
<td>1.489855</td>
</tr>
<tr>
<td>FIRSTRA</td>
<td>ADF</td>
<td>-6.49072</td>
<td>-5.43614</td>
<td>-8.86032</td>
<td>-5.30988</td>
<td>-5.92111</td>
<td>-2.34755</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>1.636163</td>
<td>1.351488</td>
<td>2.048505</td>
<td>1.442955</td>
<td>2.099199</td>
<td>1.231883</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.9403</td>
<td>1.351488</td>
<td>2.048505</td>
<td>1.442955</td>
<td>2.099199</td>
<td>1.231883</td>
</tr>
<tr>
<td>IMPERIA</td>
<td>ADF</td>
<td>-5.222</td>
<td>-4.34955</td>
<td>-5.29647</td>
<td>-4.28718</td>
<td>-4.54428</td>
<td>-2.93407</td>
</tr>
<tr>
<td></td>
<td>Durbin Watson</td>
<td>1.964126</td>
<td>1.84255</td>
<td>2.014818</td>
<td>1.824958</td>
<td>2.164767</td>
<td>1.823482</td>
</tr>
<tr>
<td></td>
<td>Crit. DF</td>
<td>-1.9415</td>
<td>1.84255</td>
<td>2.014818</td>
<td>1.824958</td>
<td>2.164767</td>
<td>1.823482</td>
</tr>
</tbody>
</table>